

# Note on Poisson cohomology on Weil bundles

B. G. R. BOSSOTO\*, O. MABIALA MIKANOU†, N. MAHOUNGOU MOUKALA‡,  
Marien NGOUABI University, Brazzaville, Congo

## Abstract

Let  $M$  be a Poisson manifold and  $A$  a Weil algebra. We describe an isomorphism of cohomology algebra and proves that Poisson cohomology with values in  $A$  is isomorphic to the tensor product of  $A$  with Poisson cohomology with real values.

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## 1 Introduction

Let  $M$  be a smooth manifold of dimension  $n$ . We denote by  $C^\infty(M)$  the algebra of smooth functions on  $M$  and by  $\mathfrak{X}(M)$  the  $C^\infty(M)$ -module of vector fields on  $M$ .

A *Poisson manifold* is a smooth manifold  $M$  for which there exists a bracket  $\{, \}$  on  $C^\infty(M)$  such that the pair  $(C^\infty(M), \{, \})$  is a real Lie algebra and for any  $f \in C^\infty(M)$ , the map

$$ad(f) : C^\infty(M) \longrightarrow C^\infty(M), g \longmapsto \{f, g\}$$

is a derivation of commutative algebra i.e

$$\{f, g \cdot h\} = \{f, g\} \cdot h + g \cdot \{f, h\}$$

for  $f, g, h \in C^\infty(M)$  [4],[5].

We denote by

$$C^\infty(M) \longrightarrow Der_{\mathbb{R}}[C^\infty(M)], f \longmapsto ad(f),$$

the adjoint representation and  $d$  the operator of cohomology associated to this representation. For any  $p \in \mathbb{N}$ ,

$$\Lambda_{Pois}^p(M) = C^p[C^\infty(M), C^\infty(M)]$$

denotes the  $C^\infty(M)$ -module of skew-symmetric multilinear forms of degree  $p$  from  $C^\infty(M)$  into  $C^\infty(M)$ . We have

$$\Lambda_{Pois}^0(M) = C^\infty(M).$$

A *Weil algebra* is a finite-dimensional associative commutative  $\mathbb{R}$ -algebra with unit  $1_A$  which admits a unique maximal ideal of codimension 1 [12].

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\*bossotob@yahoo.fr

†stive.elg@gmail.com

‡nmahomouk@yahoo.fr

Let  $A$  be a Weil algebra and  $\mathfrak{m}$  be its maximal ideal. We have  $A = \mathbb{R} \oplus \mathfrak{m}$ . The first projection

$$A = \mathbb{R} \oplus \mathfrak{m} \longrightarrow \mathbb{R}$$

is a homomorphism of algebra which is surjective, called augmentation and the unique none-zero integer  $k \in \mathbb{N}$  such that  $\mathfrak{m}^k \neq (0)$  and  $\mathfrak{m}^{k+1} = (0)$  is the height of  $A$ , where  $\mathfrak{m}^k$  denote the  $k$ th power of  $\mathfrak{m}$ .

If  $M$  is a smooth manifold, and  $A$  a Weil algebra of maximal ideal  $\mathfrak{m}$ , an infinitely near point of  $x \in M$  of kind  $A$  is a homomorphism of algebras

$$\xi : C^\infty(M) \longrightarrow A$$

such that  $[\xi(f) - f(x)] \in \mathfrak{m}$  for any  $f \in C^\infty(M)$  i.e the real part of  $\xi(f)$  is exactly  $f(x)$ .

We denote by  $M_x^A$  the set of near point of  $x$  of kind  $A$ . The set  $M^A = \bigcup_{x \in M} M_x^A$  of near points of  $M$  of kind  $A$  is a smooth manifold of dimension  $n \times \dim A$ , called manifold of infinitely near points of  $M$  of kind  $A$  [12],[6].

When both  $M$  and  $N$  are smooth manifolds and when  $h : M \longrightarrow N$  is a differentiable map, then the map

$$h^A : M^A \longrightarrow N^A, \xi \longmapsto h^A(\xi),$$

such that, for any  $g \in C^\infty(N)$ ,  $[h^A(\xi)](g) = \xi(g \circ h)$  is also differentiable.

When  $h$  is a diffeomorphism, it is the same for  $h^A$ .

Moreover, if  $\varphi : A \longrightarrow B$  is a homomorphism of Weil algebras, for any smooth manifold  $M$ , the map

$$\varphi_M : M^A \longrightarrow M^B, \xi \longmapsto \varphi \circ \xi$$

is differentiable. In particular, the augmentation  $A \longrightarrow \mathbb{R}$  defines for any smooth manifold  $M$ , the projection

$$\pi_M : M^A \longrightarrow M,$$

which assigns every infinitely near point of  $x \in M$  to its origin  $x$ . Thus  $(M^A, \pi_M, M)$  defines the bundle of infinitely near points or simply weil bundle [?],[3],[12],[6].

If  $(U, \varphi)$  is a local chart of  $M$  with coordinate functions  $(x_1, x_2, \dots, x_n)$ , the map

$$U^A \longrightarrow A^n, \xi \longmapsto (\xi(x_1), \xi(x_2), \dots, \xi(x_n)),$$

is a bijection from  $U^A$  into an open of  $A^n$ . The manifold  $M^A$  is a smooth manifold modeled over  $A^n$ , that is to say an  $A$ -manifold of dimension  $n$  [1],[10].

The set,  $C^\infty(M^A, A)$  of differentiable functions on  $M^A$  with values in  $A$  is a commutative, unitary algebra over  $A$ . When one identifies  $\mathbb{R}^A = \mathbb{R} \otimes A$  with  $A$ , for  $f \in C^\infty(M)$ , the map

$$f^A : M^A \longrightarrow A, \xi \longmapsto \xi(f)$$

is differentiable. Moreover the map

$$C^\infty(M) \longrightarrow C^\infty(M^A, A), f \longmapsto f^A,$$

is an injective homomorphism of algebras and we have for  $\lambda \in \mathbb{R}$ ,  $f, g \in C^\infty(M)$ ,

1.  $(f + g)^A = f^A + g^A$ ;
2.  $(\lambda \cdot f)^A = \lambda \cdot f^A$ ;
3.  $(f \cdot g)^A = f^A \cdot g^A$ .

## 1.1 Vector fields on Weil bundles

When  $M$  is a smooth manifold and  $A$  a Weil algebra, we denote by  $C^\infty(M^A)$  the algebra of smooth functions (with values in  $\mathbb{R}$ ) and  $\mathfrak{X}(M^A)$  the set of all vector fields on  $M^A$ . According to [2],[7], the following assertions are equivalent:

1. A vector field on  $M^A$  is a derivation of  $C^\infty(M^A)$ ;
2. A vector field on  $M^A$  is a derivation of  $C^\infty(M^A, A)$ , which are  $A$ -linear;
3. A vector field on  $M^A$  is a derivation  $X : C^\infty(M) \longrightarrow C^\infty(M^A, A)$  which verifies

$$X(fg) = X(f) \cdot g^A + f^A \cdot X(g) \text{ for any } f, g \in C^\infty(M)$$

Thus, the set  $\mathfrak{X}(M^A)$  of all vector fields on  $M^A$  is a  $C^\infty(M^A, A)$ -module.

In the following, we denote by  $Der_A[C^\infty(M^A, A)]$  the set of  $A$ -linear maps

$$X : C^\infty(M^A, A) \longrightarrow C^\infty(M^A, A)$$

such that

$$X(\varphi \cdot \psi) = X(\varphi) \cdot \psi + \varphi \cdot X(\psi), \quad \text{for any } \varphi, \psi \in C^\infty(M^A, A).$$

Thus,

$$\mathfrak{X}(M^A) = Der_A[C^\infty(M^A, A)].$$

**Remark 1.** [2] We have:

$$X(C^\infty(M^A)) \subset C^\infty(M^A) \text{ i.e } X(F) \in C^\infty(M^A) \text{ for any } F \in C^\infty(M^A).$$

The map

$$\mathfrak{X}(M^A) \times \mathfrak{X}(M^A) \longrightarrow \mathfrak{X}(M^A), (X, Y) \longmapsto [X, Y] = X \circ Y - Y \circ X$$

is skew-symmetric  $A$ -bilinear and defines a structure of an  $A$ -Lie algebra over  $\mathfrak{X}(M^A)$ .

When  $\theta : C^\infty(M) \longrightarrow C^\infty(M)$  is a vector field on  $M$ , the map

$$\theta^A : C^\infty(M) \longrightarrow C^\infty(M^A, A), f \longmapsto [\theta(f)]^A,$$

is a vector field on  $M^A$  called prolongation to  $M^A$  of the vector field  $\theta$ .

When  $A$  is a Weil algebra and if  $M$  is a Poisson manifold, the algebra  $C^\infty(M^A, A)$  of functions on  $M^A$  with values in  $A$ , is a Poisson algebra over  $A$  that is to say the Poisson bracket on  $C^\infty(M^A, A)$  is  $A$ -bilinear. In this case  $M^A$  is a  $A$ -Poisson manifold[1]. We will show in what follows that  $M^A$  admits a (real) Poisson structure i.e the algebra  $C^\infty(M^A)$  is a Poisson algebra.

In [8], it has been described two representations and therefore two cohomologies associated with these representations. We go to the following, show that these are the same cohomology. We denote by  $H_{Pois}(M^A, A)$  this algebra cohomology. In addition, we show that if  $H_{Pois}(M^A)$  denotes Poisson cohomology on  $M^A$  with real values, that is to say, defined by the Poisson algebra  $C^\infty(M^A)$ , then we have an isomorphism between  $H_{Pois}(M^A, A)$  and  $A \otimes H_{Pois}(M^A)$ .

## 2 Poisson structure on Weil bundles

In the following,  $M$  is a Poisson manifold. It is known that in this case, the  $A$ -algebra  $C^\infty(M^A, A)$  is a Poisson algebra over  $A$  i.e there exists a bracket  $\{, \}$  on  $C^\infty(M^A, A)$  such that the pair  $(C^\infty(M^A, A), \{, \})$  is a Lie algebra over  $A$  satisfying

$$\{\varphi_1 \cdot \varphi_2, \varphi_3\} = \{\varphi_1, \varphi_3\} \cdot \varphi_2 + \varphi_1 \cdot \{\varphi_2, \varphi_3\}$$

for any  $\varphi_1, \varphi_2, \varphi_3 \in C^\infty(M^A, A)$  [1].

Since  $M$  is a Poisson manifold with bracket  $\{, \}$ , the map

$$ad(f) : C^\infty(M) \longrightarrow C^\infty(M), g \longmapsto \{f, g\}$$

is a vector field on  $M$ . For any  $f \in C^\infty(M)$ , let

$$[ad(f)]^A : C^\infty(M) \longrightarrow C^\infty(M^A, A), g \longmapsto \{f, g\}^A,$$

be the prolongation of the vector field  $ad(f)$  and let

$$[\widetilde{ad(f)}]^A : C^\infty(M^A, A) \longrightarrow C^\infty(M^A, A)$$

be the unique  $A$ -linear vector field such that

$$[\widetilde{ad(f)}]^A(g^A) = [ad(f)]^A(g) = \{f, g\}^A$$

for any  $g \in C^\infty(M)$ .

According [1], for  $\varphi \in C^\infty(M^A, A)$ , the application

$$\tau_\varphi : C^\infty(M) \longrightarrow C^\infty(M^A, A), f \longmapsto -[\widetilde{ad(f)}]^A(\varphi)$$

is a vector field on  $M^A$  considered as derivation from  $C^\infty(M)$  into  $C^\infty(M^A, A)$ . It allows to construct the unique  $A$ -linear vector field on  $M^A$ ,

$$\widetilde{\tau}_\varphi : C^\infty(M^A, A) \longrightarrow C^\infty(M^A, A)$$

such that

$$\widetilde{\tau}_\varphi(f^A) = \tau_\varphi(f) \text{ for any } f \in C^\infty(M).$$

We have for  $\varphi, \psi \in C^\infty(M^A, A)$  and for  $a \in A$ ,

$$\widetilde{\tau}_{\varphi+\psi} = \widetilde{\tau}_\varphi + \widetilde{\tau}_\psi; \quad \widetilde{\tau}_{a \cdot \varphi} = a \cdot \widetilde{\tau}_\varphi; \quad \widetilde{\tau}_{\varphi \cdot \psi} = \varphi \cdot \widetilde{\tau}_\psi + \psi \cdot \widetilde{\tau}_\varphi.$$

For any  $\varphi, \psi \in C^\infty(M^A, A)$ , asking

$$\{\varphi, \psi\}_A = \widetilde{\tau}_\varphi(\psi),$$

the map

$$\{, \}_A : C^\infty(M^A, A) \times C^\infty(M^A, A) \longrightarrow C^\infty(M^A, A), (\varphi, \psi) \longmapsto \{\varphi, \psi\}_A$$

defines a structure of  $A$ -Poisson algebra on  $C^\infty(M^A, A)$ [1]. Hence the following theorem:

**Theorem 1.** [1] *If  $M$  is a Poisson manifold with bracket  $\{, \}$ , then  $\{, \}_A$  is the prolongation on  $M^A$  of the structure of Poisson on  $M$  defined by  $\{, \}$ .*

## 2.1 Cohomology associated with a Poisson structure on Weil bundles

When  $M$  is a Poisson manifold with bracket  $\{, \}$ , the map

$$\tau : C^\infty(M) \longrightarrow \text{Der}_A[C^\infty(M^A, A)], f \longmapsto -[\widetilde{ad(f)}]^A$$

is a representation of  $C^\infty(M)$  into  $C^\infty(M^A, A)$ . We denote by  $\widetilde{d}$  the operator of cohomology associated to this representation.

For any  $p \in \mathbb{N}$ ,

$$\Lambda_{Pois}^p(M^A, \sim) = C^p[C^\infty(M), C^\infty(M^A, A)]$$

denotes the  $C^\infty(M^A, A)$ -module of skew-symmetric multilinear forms of degree  $p$  from  $C^\infty(M)$  into  $C^\infty(M^A, A)$ . We have

$$\Lambda_{Pois}^0(M^A, \sim) = C^\infty(M^A, A).$$

We denote

$$\Lambda_{Pois}(M^A, \sim) = \bigoplus_{p=0}^n \Lambda_{Pois}^p(M^A, \sim).$$

Thus, for  $\Omega \in \Lambda_{Pois}^p(M^A, \sim)$  and  $f_1, \dots, f_{p+1} \in C^\infty(M)$ , we have

$$\begin{aligned} \widetilde{d}\Omega(f_1, \dots, f_{p+1}) &= \sum_{i=1}^{p+1} (-1)^i [\widetilde{ad(f_i)}]^A [\Omega(f_1, \dots, \widehat{f_i}, \dots, f_{p+1})] \\ &+ \sum_{1 \leq i < j \leq p+1} (-1)^{i+j} \Omega(\{f_i, f_j\}, f_1, \dots, \widehat{f_i}, \dots, \widehat{f_j}, \dots, f_{p+1}) \end{aligned}$$

where  $\widehat{f_i}$  means that the term  $f_i$  is omitted.

When  $\eta \in \Lambda_{Pois}^p(M)$ , then

$$\eta^A : C^\infty(M) \times \dots \times C^\infty(M) \longrightarrow C^\infty(M^A, A), (f_1, \dots, f_p) \longmapsto [\eta(f_1, \dots, f_p)]^A$$

is skew-symmetric multilinear forms of degree  $p$  from  $C^\infty(M)$  into  $C^\infty(M^A, A)$  i.e

$$\eta^A \in \Lambda_{Pois}^p(M^A, \sim).$$

We denote by  $H_{Pois}(M^A, \sim)$  the associated algebra cohomology.

Similary, the map

$$C^\infty(M^A, A) \longrightarrow \text{Der}_A[C^\infty(M^A, A)], \varphi \longmapsto \widetilde{\tau}_\varphi,$$

is a representation of  $C^\infty(M^A, A)$  into  $C^\infty(M^A, A)$ . We denote by  $\widetilde{d}_A$  the cohomology operator associated to this representation.

For any  $p \in \mathbb{N}$ ,

$$\Lambda_{Pois}^p(M^A, \sim_A) = C^p[C^\infty(M^A, A), C^\infty(M^A, A)]$$

denotes the  $C^\infty(M^A, A)$ -module of skew-symmetric multilinear forms of degree  $p$  from  $C^\infty(M^A, A)$  into  $C^\infty(M^A, A)$ . We have

$$\Lambda_{Pois}^0(M^A, \sim_A) = C^\infty(M^A, A).$$

We denote

$$\Lambda_{Pois}(M^A, \sim_A) = \bigoplus_{p=0}^n \Lambda_{Pois}^p(M^A, \sim_A).$$

For  $\Omega \in \Lambda_{Pois}^p(M^A, \sim_A)$  and  $\varphi_1, \varphi_2, \dots, \varphi_{p+1} \in C^\infty(M^A, A)$ , we have

$$\begin{aligned} \widetilde{d}_A \Omega(\varphi_1, \dots, \varphi_{p+1}) &= \sum_{i=1}^{p+1} (-1)^{i-1} \widetilde{\tau}_{\varphi_i} [\Omega(\varphi_1, \dots, \widehat{\varphi_i}, \dots, \varphi_{p+1})] \\ &+ \sum_{1 \leq i < j \leq p+1} (-1)^{i+j} \Omega(\{\varphi_i, \varphi_j\}_A, \varphi_1, \dots, \widehat{\varphi_i}, \dots, \widehat{\varphi_j}, \dots, \varphi_{p+1}) \end{aligned}$$

i.e

$$\begin{aligned} \widetilde{d}_A \Omega(\varphi_1, \varphi_2, \dots, \varphi_{p+1}) &= \sum_{i=1}^{p+1} (-1)^{i-1} \{\varphi_i, \Omega(\varphi_1, \dots, \widehat{\varphi_i}, \dots, \varphi_{p+1})\}_A \\ &+ \sum_{1 \leq i < j \leq p+1} (-1)^{i+j} \Omega(\{\varphi_i, \varphi_j\}_A, \varphi_1, \dots, \widehat{\varphi_i}, \dots, \widehat{\varphi_j}, \dots, \varphi_{p+1}). \end{aligned}$$

We denote by  $H_{Pois}(M^A, \sim_A)$  the algebra cohomology associated with the representation

$$\widetilde{\tau} : C^\infty(M^A, A) \longrightarrow Der_A[C^\infty(M^A, A)], \varphi \longmapsto \widetilde{\tau}_\varphi.$$

We have the following result:

**Theorem 2.** *The Poisson algebra cohomology  $H_{Pois}(M^A, \sim)$  and  $H_{Pois}(M^A, \sim_A)$  are identical.*

*Proof.* When  $M$  is a Poisson manifold with bracket  $\{, \}$ , the map

$$\tau : C^\infty(M) \longrightarrow Der_A[C^\infty(M^A, A)], f \longmapsto -[ad(\widetilde{f})]^A$$

is a representation of  $C^\infty(M)$  into  $C^\infty(M^A, A)$ . It helps to construct the representation

$$\widetilde{\tau} : C^\infty(M^A, A) \longrightarrow Der_A[C^\infty(M^A, A)], \varphi \longmapsto \widetilde{\tau}_\varphi$$

such that  $\widetilde{\tau}(f^A) = \tau(f)$  for every  $f \in C^\infty(M)$ .

If

$$\rho : C^\infty(M^A, A) \longrightarrow Der_A[C^\infty(M^A, A)],$$

is another representation such that  $H(f^A) = \tau(f)$  for every  $f \in C^\infty(M)$ , then  $\widetilde{\tau}(f^A) = \rho(f^A)$ . Therefore  $\widetilde{\tau} = \rho[1]$ .

Recipocally, the repr sentation

$$\widetilde{\tau} : C^\infty(M^A, A) \longrightarrow Der_A[C^\infty(M^A, A)], \varphi \longmapsto \widetilde{\tau}_\varphi,$$

allows to construct the map

$$C^\infty(M) \xrightarrow{T^A} C^\infty(M^A, A) \xrightarrow{\widetilde{\tau}} Der_A[C^\infty(M^A, A)C^\infty(M^A, A)], f \longmapsto f^A \longmapsto \widetilde{\tau}(f^A) = -[ad(\widetilde{f})]^A$$

which is the representation  $\tau$ . Representations  $\tau$  and  $\widetilde{\tau}$  are isomorphic and therefore, cohomology algebras  $H_{Pois}(M^A, \sim_A)$  and  $H_{Pois}(M^A, \sim)$  are identical.  $\square$

In the following, this cohomology is simply denoted by  $H_{Pois}(M^A, A)$  and called cohomology associated with a  $A$ -Poisson algebra  $C^\infty(M^A, A)$ .

In what follows, we will show that  $M^A$  is a Poisson manifold and establish the following theorem:

**Theorem 3.** *If  $(M, \{, \})$  is a Poisson manifold, then  $M^A$  is a Poisson manifold. Moreover Poisson structure  $\{, \}_\mathbb{R}$  defined on  $M^A$  is the restriction to  $C^\infty(M^A) \times C^\infty(M^A)$  of prolongation to  $M^A$  of Poisson structure  $\{, \}_A$  i.e  $\{, \}_\mathbb{R} = \{, \}_A|_{C^\infty(M^A) \times C^\infty(M^A)}$ . Moreover, if  $H_{Pois}(M^A)$  denote this associated Poisson cohomology, then  $H_{Pois}(M^A, A)$  and  $A \otimes H_{Pois}(M^A)$  are isomorphic.*

*Proof.* Since  $\widetilde{\tau}_\varphi : C^\infty(M^A, A) \rightarrow C^\infty(M^A, A)$  is a vector field which is  $A$ -linear, then  $\widetilde{\tau}_\varphi [C^\infty(M^A)] \subset C^\infty(M^A)$  i.e  $\widetilde{\tau}_\varphi(G) \in C^\infty(M^A)$  for all  $G \in C^\infty(M^A)$ . Consequently,  $\widetilde{\tau}_F(G) = \{F, G\}_A \in C^\infty(M^A)$  for  $F, G \in C^\infty(M^A)$ .

The map

$$\{, \}_A|_{C^\infty(M^A) \times C^\infty(M^A)} : C^\infty(M^A) \times C^\infty(M^A) \rightarrow C^\infty(M^A), (F, G) \mapsto \{F, G\}_A$$

is well defined.

Other properties that arise from  $C^\infty(M^A) \subset C^\infty(M^A, A)$ . The bracket  $\{, \}_A|_{C^\infty(M^A) \times C^\infty(M^A)}$  is a Poisson bracket on  $C^\infty(M^A)$ .

Let

$$\sigma : C^\infty(M^A, A) \rightarrow A \otimes C^\infty(M^A),$$

be the canonical isomorphism. All Poisson form of degree  $p$ ,

$$\eta : C^\infty(M^A, A) \times \dots \times C^\infty(M^A, A) \rightarrow C^\infty(M^A, A)$$

is equivalent to the given of the  $p$ -form

$$A \otimes C^\infty(M^A) \times \dots \times A \otimes C^\infty(M^A) \rightarrow A \otimes C^\infty(M^A)$$

i.e of the form

$$\omega : A \otimes [C^\infty(M^A) \times \dots \times C^\infty(M^A)] \rightarrow A \otimes C^\infty(M^A)$$

By passing to the quotient, we have the isomorphism

$$H_{Pois}(M^A, A) \rightarrow A \otimes H_{Pois}(M^A), \bar{\eta} \mapsto \overline{\omega = \sigma \circ \eta \circ (\sigma^{-1} \times \dots \times \sigma^{-1})}.$$

That ends Proof. □

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